

CLAIMS

1. A system estimation method for making state estimation
robust and optimizing a forgetting factor ρ simultaneously in an
5 estimation algorithm, in which

for a state space model expressed by following expressions:

$$x_{k+1} = F_k x_k + G_k w_k$$

$$y_k = H_k x_k + v_k$$

$$z_k = H_k x_k$$

10 here,

x_k : a state vector or simply a state,

w_k : a system noise,

v_k : an observation noise,

y_k : an observation signal,

15 z_k : an output signal,

F_k : dynamics of a system, and

G_k : a drive matrix,

a maximum energy gain to a filter error from a disturbance
weighted with the forgetting factor ρ as an evaluation criterion
20 is suppressed to be smaller than a term corresponding to a
previously given upper limit value γ_f , and

the system estimation method comprises:

a step at which a processing section inputs the upper limit
value γ_f , the observation signal y_k as an input of a filter and
25 a value including an observation matrix H_k from a storage section
or an input section;

a step at which the processing section determines the
forgetting factor ρ relevant to the state space model in accordance
with the upper limit value γ_f ;

30 a step at which the processing section reads out an initial
value or a value including the observation matrix H_k at a time from
the storage section and uses the forgetting factor ρ to execute

a hyper H_∞ filter expressed by a following expression:

$$\hat{x}_{k|k} = F_{k-1}\hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k F_{k-1}\hat{x}_{k-1|k-1})$$

here,

$\hat{x}_{k|k}$; an estimated value of a state x_k at a time k using observation
5 signals y_0 to y_k ,

$K_{s,k}$; a filter gain,

a step at which the processing section stores an obtained
value relating to the hyper H_∞ filter into the storage section;

a step at which the processing section calculates an
10 existence condition based on the upper limit value γ_f and the
forgetting factor ρ by the obtained observation matrix H_i or the
observation matrix H_i and the filter gain $K_{s,i}$, and

a step at which the processing section sets the upper limit
value to be small within a range where the existence condition
15 is satisfied at each time and stores the value into the storage
section, by decreasing the upper limit value γ_f and repeating the
step of executing the hyper H_∞ filter.

2. The system estimation method according to claim 1,
20 wherein the processing section calculates the existence condition
in accordance with a following expression:

$$\hat{\Sigma}_{i|i}^{-1} = \hat{\Sigma}_{i|i-1}^{-1} + \frac{1 - \gamma_f^{-2}}{\rho} H_i^T H_i > 0, \quad i = 0, \dots, k \quad (17)$$

3. The system estimation method according to claim 1,
25 wherein the processing section calculates the existence condition
in accordance with a following expression:

$$-\varrho \hat{\Xi}_i + \rho \gamma_f^2 > 0, \quad i = 0, \dots, k \quad (18)$$

here,

$$\varrho = 1 - \gamma_f^2, \quad \hat{\Xi}_i = \frac{\rho H_i K_{s,i}}{1 - H_i K_{s,i}}, \quad \rho = 1 - \chi(\gamma_f) \quad (19)$$

4. The system estimation method according to claim 1,
wherein the forgetting factor ρ and the upper limit value γ_f have
5 a following relation:

$0 < \rho = 1 - \chi(\gamma_f) \leq 1$, where $\chi(\gamma_f)$ denotes a monotonically damping
function of γ_f to satisfy $\chi(1) = 1$ and $\chi(\infty) = 0$.

5. The system estimation method according to claim 1,
10 wherein at the step of executing the hyper H_∞ filter,
the processing section obtains the filter gain $K_{s,k}$ by
following expressions:

$$\tilde{z}_{k|k} = H_k \hat{x}_{k|k} \quad (10)$$

$$\hat{x}_{k|k} = F_{k-1} \hat{x}_{k-1|k-1} + K_{s,k} (y_k - H_k F_{k-1} \hat{x}_{k-1|k-1}) \quad (11)$$

$$K_{s,k} = \hat{\Sigma}_{k|k-1} H_k^T (H_k \hat{\Sigma}_{k|k-1} H_k^T + \rho)^{-1} \quad (12)$$

$$\left. \begin{aligned} \hat{\Sigma}_{k|k} &= \hat{\Sigma}_{k|k-1} - \hat{\Sigma}_{k|k-1} C_k^T R_{e,k}^{-1} C_k \hat{\Sigma}_{k|k-1} \\ \hat{\Sigma}_{k+1|k} &= (F_k \hat{\Sigma}_{k|k} F_k^T) / \rho \end{aligned} \right\} \quad (13)$$

here,

$$\begin{aligned} e_{f,i} &= \tilde{z}_{i|i} - H_i x_i, \quad \hat{x}_{0|0} = \tilde{x}_0, \quad \hat{\Sigma}_{1|0} = \Sigma_0 \\ R_{e,k} &= R_k + C_k \hat{\Sigma}_{k|k-1} C_k^T, \quad R_k = \begin{bmatrix} \rho & 0 \\ 0 & -\rho \gamma_f^2 \end{bmatrix}, \quad C_k = \begin{bmatrix} H_k \\ H_k \end{bmatrix} \end{aligned} \quad (14)$$

$$0 < \rho = 1 - \chi(\gamma_f) \leq 1, \quad \gamma_f > 1 \quad (15)$$

$$G_k G_k^T = \frac{\chi(\gamma_f)}{\rho} F_k \hat{\Sigma}_{k|k} F_k^T \quad (16)$$

wherein a right side of the expression (16) can be more
15 generalized,

here,

x_k : the state vector or simply the state,

y_k : the observation signal,

z_k : the output signal,

5 F_k : the dynamics of the system,

H_k : the observation matrix,

$\hat{x}_{k|k}$: the estimated value of the state x_k at the time k using the observation signals y_0 to y_k ,

$\hat{\Sigma}_{k|k}$: corresponding to a covariance matrix of an error of $\hat{x}_{k|k}$,

10 $K_{s,k}$: the filter gain,

$e_{f,j}$: the filter error, and

$R_{e,k}$: an auxiliary variable.

6. The system estimation method according to claim 5,
15 wherein the step of executing the hyper H_∞ filter includes:

a step at which the processing section calculates the filter gain $K_{s,k}$ by using the expression (12) based on an initial condition;

a step at which the processing section updates a filter equation of the H_∞ filter of the expression (11);

20 a step at which the processing section calculates $\hat{\Sigma}_{k|k}$ and $\hat{\Sigma}_{k+1|k}$ by using the expression (13); and

a step at which the processing section repeatedly executes the respective steps while advancing the time k .

25 7. The system estimation method according to claim 1, wherein at the step of executing the hyper H_∞ filter,

the processing section calculates the filter gain $K_{s,k}$ by using a gain matrix K_k and by following expressions:

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k \hat{x}_{k-1|k-1}) \quad (20)$$

$$K_{s,k} = K_k(:, 1)/R_{e,k}(1, 1), \quad K_k = \rho^{\frac{1}{2}}(\rho^{-\frac{1}{2}}K_k R_{e,k}^{-\frac{1}{2}} J_1^{-1}) J_1 R_{e,k}^{\frac{1}{2}} \quad (21)$$

$$\left[\begin{array}{c|c} R_k^{\frac{1}{2}} & C_k \hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \\ \hline 0 & \rho^{-\frac{1}{2}} \hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \end{array} \right] \Theta(k) = \left[\begin{array}{c|c} R_{e,k}^{\frac{1}{2}} & 0 \\ \hline \rho^{-\frac{1}{2}} K_k R_{e,k}^{-\frac{1}{2}} J_1^{-1} & \hat{\Sigma}_{k+1|k}^{\frac{1}{2}} \end{array} \right] \quad (22)$$

Where,

$$\begin{aligned} R_k &= R_k^{\frac{1}{2}} J_1 R_k^{\frac{1}{2}}, \quad R_k^{\frac{1}{2}} = \begin{bmatrix} \rho^{\frac{1}{2}} & 0 \\ 0 & \rho^{\frac{1}{2}} \gamma_f \end{bmatrix}, \quad J_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \hat{\Sigma}_{k|k-1} = \hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \\ R_{e,k} &= R_k + C_k \hat{\Sigma}_{k|k-1} C_k^T, \quad C_k = \begin{bmatrix} H_k \\ H_k \end{bmatrix}, \quad R_{e,k} = R_{e,k}^{\frac{1}{2}} J_1 R_{e,k}^{\frac{1}{2}}, \quad \hat{x}_{0|0} = \hat{x}_0 \end{aligned} \quad (23)$$

$\Theta(k)$ denotes a J-unitary matrix, that is, satisfies $\Theta(k) J \Theta(k)^T = J$, $J = (J_1 \oplus I)$, I denotes a unit matrix, $K_k(:, 1)$ denotes a column vector of a first column of the matrix K_k ,

5 wherein J_1^{-1} and J_1 can be deleted in the expressions (21)

and (22),

here,

$\hat{x}_{k|k}$: the estimated value of the state x_k at the time k using the observation signals y_0 to y_k ,

10 y_k : the observation signal,

F_k : the dynamics of the system,

$K_{s,k}$: the filter gain,

H_k : the observation matrix,

$\hat{\Sigma}_{k|k}$: corresponding to a covariance matrix of an error of $\hat{x}_{k|k}$,

15 $\Theta(k)$: the J-unitary matrix, and

$R_{e,k}$: an auxiliary variable.

8. The system estimation method according to claim 7, wherein the step of executing the hyper H_∞ filter includes:

20 a step at which the processing section calculates K_k and

$\hat{\Sigma}_{k+1|k}^{1/2}$ by using the expression (22);

a step at which the processing section calculates the filter gain $K_{s,k}$ based on the initial condition and by using the expression (21);

5 a step at which the processing section updates a filter equation of the H_∞ filter of the expression (20); and

a step at which the processing section repeatedly executes the respective steps while advancing the time k .

10 9. The system estimation method according to claim 1, wherein at the step of executing the hyper H_∞ filter,

the processing section obtains the filter gain $K_{s,k}$ by using a gain matrix K_k and by following expressions:

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + \bar{K}_{s,k}(y_k - H_k \hat{x}_{k-1|k-1}) \quad (61)$$

$$\bar{K}_{s,k} = \bar{K}_k(:, 1)/R_{e,k}(1, 1), \quad \bar{K}_k = \rho^{\frac{1}{2}}(\bar{K}_k R_{e,k}^{-\frac{1}{2}})R_{e,k}^{\frac{1}{2}} \quad (62)$$

$$\begin{bmatrix} R_{e,k+1}^{\frac{1}{2}} & 0 \\ \left[\begin{array}{c} \bar{K}_{k+1} \\ 0 \end{array} \right] R_{e,k+1}^{-\frac{1}{2}} J_1 & \tilde{L}_{k+1} R_{r,k+1}^{-\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} R_{e,k}^{\frac{1}{2}} & \check{C}_{k+1} \tilde{L}_k R_{r,k}^{-\frac{1}{2}} \\ \left[\begin{array}{c} 0 \\ \bar{K}_k \end{array} \right] R_{e,k}^{-\frac{1}{2}} J_1 & \rho^{-\frac{1}{2}} \tilde{L}_k R_{r,k}^{-\frac{1}{2}} \end{bmatrix} \Theta(k) \quad (63)$$

15 here, $\Theta(k)$ denotes an arbitrary J -unitary matrix, and $\check{C}_k = \check{C}_{k+1}\Psi$ is established,

where

$$\begin{aligned} R_k &= R_k^{\frac{1}{2}} J_1 R_k^{\frac{1}{2}}, \quad R_k^{\frac{1}{2}} = \begin{bmatrix} \rho^{\frac{1}{2}} & 0 \\ 0 & \rho^{\frac{1}{2}} \gamma_f \end{bmatrix}, \quad J_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \hat{\Sigma}_{k|k-1} = \hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \hat{\Sigma}_{k|k-1}^{\frac{1}{2}} \\ R_{e,k} &= R_k + C_k \hat{\Sigma}_{k|k-1} C_k^T, \quad C_k = \begin{bmatrix} H_k \\ H_k \end{bmatrix}, \quad R_{e,k} = R_{e,k}^{\frac{1}{2}} J_1 R_{e,k}^{\frac{1}{2}}, \quad \hat{x}_{0|0} = \hat{x}_0 \end{aligned} \quad (23)$$

here,

20 $\hat{x}_{k|k}$: the estimated value of the state x_k at the time k using the observation signals y_0 to y_k ,

y_k : the observation signal,

$K_{s,k}$: the filter gain,

H_k : the observation matrix,

$\Theta(k)$: the J-unitary matrix, and
 $R_{e,k}$: an auxiliary variable.

10. The system estimation method according to claim 9,
5 wherein the step of executing the hyper H_∞ filter includes:
a step at which the processing section calculates K_k^- by using
the expression (63);
a step at which the processing section calculates the filter
gain $K_{s,k}^-$ based on the initial condition and by using the expression
10 (62);
a step at which the processing section updates a filter
equation of the H_∞ filter of the expression (61); and
a step at which the processing section repeatedly executes
the respective steps while advancing the time k .

15

11. The system estimation method according to claim 1,
wherein at the step of executing the hyper H_∞ filter,
the processing section obtains the filter gain $K_{s,k}$ by using
a gain matrix K_k^- and by following expressions:

$$\hat{x}_{k|k} = \hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k \hat{x}_{k-1|k-1}) \quad (25)$$

$$K_{s,k} = \rho^{\frac{1}{2}} \bar{K}_k(:, 1) / R_{e,k}(1, 1) \quad (26)$$

$$\begin{bmatrix} \bar{K}_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{K}_k \end{bmatrix} - \rho^{-\frac{1}{2}} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T \quad (27)$$

$$\tilde{L}_{k+1} = \rho^{-\frac{1}{2}} \tilde{L}_k - \begin{bmatrix} 0 \\ \bar{K}_k \end{bmatrix} R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k \quad (28)$$

$$R_{e,k+1} = R_{e,k} - \check{C}_{k+1} \tilde{L}_k R_{r,k}^{-1} \tilde{L}_k^T \check{C}_{k+1}^T \quad (29)$$

$$R_{r,k+1} = R_{r,k} - \tilde{L}_k^T \check{C}_{k+1}^T R_{e,k}^{-1} \check{C}_{k+1} \tilde{L}_k \quad (30)$$

Where,

$$\begin{aligned} \check{C}_{k+1} &= \begin{bmatrix} \check{H}_{k+1} \\ \check{H}_{k+1} \end{bmatrix}, \quad \check{H}_{k+1} = [u_{k+1} \ u(k+1-N)] = [u(k+1) \ u_k], \quad \check{H}_1 = [u(1), 0, \dots, 0] \\ R_{e,1} &= R_1 + \check{C}_1 \check{\Sigma}_{1|0} \check{C}_1^T, \quad R_1 = \begin{bmatrix} \rho & 0 \\ 0 & -\rho\gamma_f^2 \end{bmatrix}, \quad \check{\Sigma}_{1|0} = \text{diag}\{\rho^2, \rho^3, \dots, \rho^{N+2}\}, \quad \rho = 1 - \chi(\gamma_f) \\ \tilde{L}_0 &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{(N+1) \times 2}, \quad R_{r,0} = \begin{bmatrix} -1 & 0 \\ 0 & \rho^{-N} \end{bmatrix}, \quad \bar{K}_0 = 0, \quad \hat{x}_{0|0} = \hat{x}_0, \quad \bar{K}_k = \rho^{-\frac{1}{2}} K_k \end{aligned} \quad (31)$$

wherein the above expressions can be arranged also with respect to K_k instead of K_k^- ,

here,

5 y_k : the observation signal,

F_k : the dynamics of the system,

H_k : the observation matrix,

$\hat{x}_{k|k}$: the estimated value of the state x_k at the time k using the observation signals y_0 to y_k ,

10 $K_{s,k}$: the filter gain, obtained from the gain matrix K_k^- , and

$R_{e,k}$, \tilde{L}_k : an auxiliary variable.

12. The system estimation method according to claim 11, wherein the step of executing the hyper H_∞ filter includes:

15 a step at which the processing section recursively calculates K_{k+1}^- based on a previously determined initial condition

and by using the expression (27);

a step at which the processing section calculates the system gain $K_{s,k}$ by using the expression (26);

5 a step at which the processing section calculates the existence condition;

a step at which the processing section updates a filter equation of the H_∞ filter of the expression (25) when the existence condition is satisfied, and repeatedly executes the respective steps repeatedly while advancing the time k ; and

10 a step of increasing the upper limit value γ_f when the existence condition is not satisfied.

13. The system estimation method according to claim 1, wherein an estimated value $z_{k|k}^v$ of the output signal is obtained from the state estimated value $\hat{x}_{k|k}$ at the time k by a following expression:

$$z_{k|k}^v = H_k \hat{x}_{k|k}.$$

14. The system estimation method according to claim 1, wherein the H_∞ filter equation is applied to obtain the state estimated value $\hat{x}_{k|k}$,

a pseudo-echo is estimated by a following expression:

$$\hat{d}_k = \sum_{i=0}^{N-1} \hat{h}_i[k] u_{k-i}, \quad k = 0, 1, 2, \dots \quad (34)$$

and

25 an echo canceller is realized by canceling an actual echo by the obtained pseudo-echo.

15. A system estimation program for causing a computer to make state estimation robust and to optimize a forgetting factor ρ simultaneously in an estimation algorithm, in which

for a state space model expressed by following expressions:

$$x_{k+1} = F_k x_k + G_k w_k$$

$$y_k = H_k x_k + v_k$$

$$z_k = H_k x_k$$

5 here,

x_k : a state vector or simply a state,

w_k : a system noise,

v_k : an observation noise,

y_k : an observation signal,

10 z_k : an output signal,

F_k : dynamics of a system, and

G_k : a drive matrix,

a maximum energy gain to a filter error from a disturbance weighted with the forgetting factor ρ as an evaluation criterion
15 is suppressed to be smaller than a term corresponding to a previously given upper limit value γ_f , and

the system estimation program causes the computer to execute:

a step at which a processing section inputs the upper limit
20 value γ_f , the observation signal y_k as an input of a filter and a value including an observation matrix H_k from a storage section or an input section;

a step at which the processing section determines the forgetting factor ρ relevant to the state space model in accordance
25 with the upper limit value γ_f ;

a step at which the processing section reads out an initial value or a value including the observation matrix H_k at a time from the storage section and uses the forgetting factor ρ to execute a hyper H_∞ filter expressed by a following expression:

$$30 \hat{x}_{k|k} = F_{k-1} \hat{x}_{k-1|k-1} + K_{s,k} (y_k - H_k F_{k-1} \hat{x}_{k-1|k-1})$$

here,

$\hat{x}_{k|k}$; an estimated value of a state x_k at a time k using observation

signals y_0 to y_k ,
 F_k : dynamics of the system, and
 $K_{s,k}$; a filter gain,
 a step at which the processing section stores an obtained
 5 value relating to the hyper H_∞ filter into the storage section;
 a step at which the processing section calculates an
 existence condition based on the upper limit value γ_f and the
 forgetting factor ρ by the obtained observation matrix H_i or the
 observation matrix H_i and the filter gain $K_{s,i}$; and
 10 a step at which the processing section sets the upper limit
 value to be small within a range where the existence condition
 is satisfied at each time and stores the value into the storage
 section by decreasing the upper limit value γ_f and repeating the
 step of executing the hyper H_∞ filter.
 15

16. A computer readable recording medium recording a system
 estimation program for causing a computer to make state estimation
 robust and to optimize a forgetting factor ρ simultaneously in
 an estimation algorithm, in which
 20 for a state space model expressed by following expressions:

$$x_{k+1} = F_k x_k + G_k w_k$$

$$y_k = H_k x_k + v_k$$

$$z_k = H_k x_k$$
 here,
 25 x_k : a state vector or simply a state,
 w_k : a system noise,
 v_k : an observation noise,
 y_k : an observation signal,
 z_k : an output signal,
 30 F_k : dynamics of a system, and
 G_k : a drive matrix,
 a maximum energy gain to a filter error from a disturbance

weighted with the forgetting factor ρ as an evaluation criterion is suppressed to be smaller than a term corresponding to a previously given upper limit value γ_f , and

the computer readable recording medium recording the system
5 estimation program causes the computer to execute:

a step at which a processing section inputs the upper limit value γ_f , the observation signal y_k as an input of a filter and a value including an observation matrix H_k from a storage section or an input section;

10 a step at which the processing section determines the forgetting factor ρ relevant to the state space model in accordance with the upper limit value γ_f ;

a step at which the processing section reads out an initial value or a value including the observation matrix H_k at a time from
15 the storage section and uses the forgetting factor ρ to execute a hyper H_∞ filter expressed by a following expression:

$$\hat{x}_{k|k} = F_{k-1}\hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k F_{k-1}\hat{x}_{k-1|k-1})$$

here,

$\hat{x}_{k|k}$; an estimated value of a state x_k at a time k using observation
20 signals y_0 to y_k ,

F_k : dynamics of the system, and

$K_{s,k}$; a filter gain,

a step at which the processing section stores an obtained value relating to the hyper H_∞ filter into the storage section;

25 a step at which the processing section calculates an existence condition based on the upper limit value γ_f and the forgetting factor ρ by the obtained observation matrix H_i or the observation matrix H_i and the filter gain $K_{s,i}$; and

a step at which the processing section sets the upper limit
30 value to be small within a range where the existence condition is satisfied at each time and stores the value into the storage section by decreasing the upper limit value γ_f and repeating the

step of executing the hyper H_∞ filter.

17. A system estimation device for making state estimation robust and optimizing a forgetting factor ρ simultaneously in an estimation algorithm, in which

for a state space model expressed by following expressions:

$$x_{k+1} = F_k x_k + G_k w_k$$

$$y_k = H_k x_k + v_k$$

$$z_k = H_k x_k$$

here,

x_k : a state vector or simply a state,

w_k : a system noise,

v_k : an observation noise,

y_k : an observation signal,

z_k : an output signal,

F_k : dynamics of a system, and

G_k : a drive matrix,

a maximum energy gain to a filter error from a disturbance weighted with the forgetting factor ρ as an evaluation criterion is suppressed to be smaller than a term corresponding to a previously given upper limit value γ_f ,

the system estimation device comprises:

a processing section to execute the estimation algorithm;

and

a storage section to which reading and/or writing is performed by the processing section and which stores respective observed values, set values, and estimated values relevant to the state space model,

the processing section inputs the upper limit value γ_f , the observation signal y_k as an input of a filter and a value including an observation matrix H_k from a storage section or an input section,

the processing section determines the forgetting factor ρ

relevant to the state space model in accordance with the upper limit value γ_f ,

the processing section reads out an initial value or a value including the observation matrix H_k at a time from the storage section and uses the forgetting factor ρ to execute a hyper H_∞ filter expressed by a following expression:

$$\hat{x}_{k|k} = F_{k-1}\hat{x}_{k-1|k-1} + K_{s,k}(y_k - H_k F_{k-1}\hat{x}_{k-1|k-1})$$

here,

$\hat{x}_{k|k}$; an estimated value of a state x_k at a time k using observation signals y_0 to y_k ,

F_k : dynamics of the system, and

$K_{s,k}$; a filter gain,

the processing section stores an obtained value relating to the hyper H_∞ filter into the storage section,

the processing section calculates an existence condition based on the upper limit value γ_f and the forgetting factor ρ by the obtained observation matrix H_i or the observation matrix H_i and the filter gain $K_{s,i}$, and

the processing part sets the upper limit value to be small within a range where the existence condition is satisfied at each time and stores the value into the storage section by decreasing the upper limit value γ_f and repeating the step of executing the hyper H_∞ filter.